Memory Properties of a Separation Network

Paweł Matykiewicz

Ph. D. Student

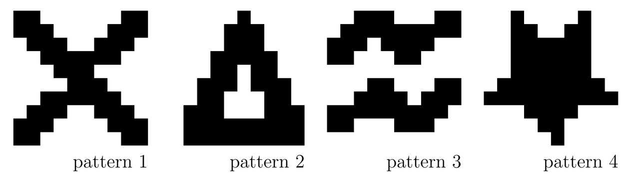
Department of Informatics

Nicholaus Copernicus University

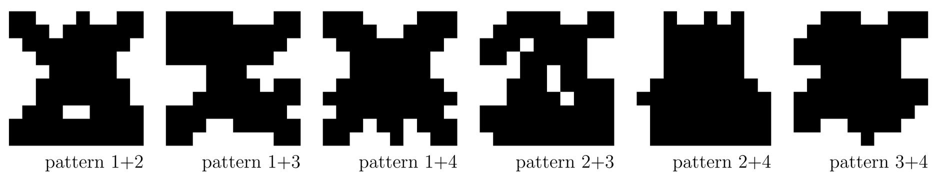
www.neuron.m4u.pl

`pawelm@phys.uni.torun.pl`

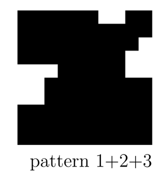
Stored Patterns



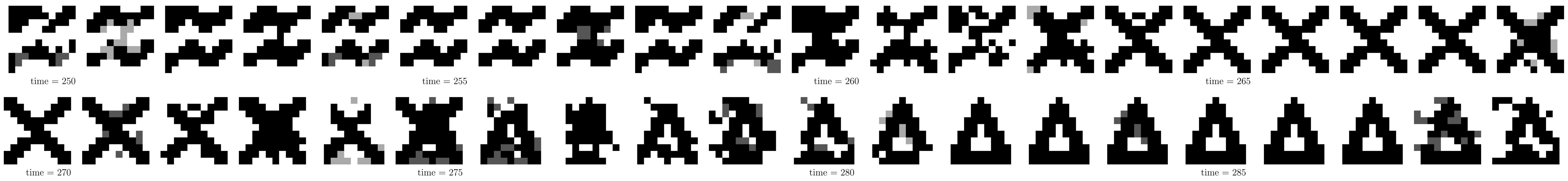
Mixed Patterns



Input Pattern (mixed)



Output Patterns



Abstract

Chaotic neural network with external inputs has been used as a mixed input pattern separator. Although highly chaotic dynamical system (LLE ≈ 0.6) is applied here the network is capable to retrieve stored patterns. The idea is based on a "dynamical mapping" scheme as an effective framework for cortical mapping. This feature allows for a more effective pattern retrieval and separation by the network.

Network Model

Following equations define an auto-associative chaotic memory:

$$\eta_i(t+1) = k_a \eta_i(t) + \sum_{w_{ij} \in W_i^E} w_{ij} x_j(t) + e_i, \qquad (1)$$

$$\zeta_i(t+1) = k_r \zeta_i(t) - \alpha x_i(t) + + \sum_{w_{ij} \in W_i^I} w_{ij} x_j(t) + \theta, \qquad (2)$$

$$x_i(t+1) = f\{\eta_i(t+1) + \zeta_i(t+1)\}. \qquad (3)$$

$$f(u) = \frac{1}{1+\exp(\frac{-u}{\varepsilon})}. \qquad (4)$$

- $x_i(t+1)$ - an output of the ith neuron at the time $t+1$
- f - a continuous output function
- $\eta_i(t)$ - all positive post-synaptical potentials (*pseudo*-action potential)
- $\zeta_i(t)$ - all negative potentials (resting potential)
- k_r - a decay parameters for resting potential
- k_a - a decay parameters for *pseudo*-action potential
- e_i - a strength of the external input applied to the ith neuron
- α - a refractory scaling parameter
- θ - a threshold value independent of the serial number i of a neuron
- ε - a steepness parameter of the continuous output function f

The sets W_i^E and W_i^I consists excitatory and inhibitory weights, respectively. They have been obtained from the Hebbian learning rule:

$$w_{ij} = \frac{1}{n} \sum_{p=1}^{m} (2x_i^p - 1)(2x_j^p - 1), \qquad (5)$$

- x^p - the memorized patterns
- m - the number of patterns (here $m = 4$)
- n - the number of synchronously updated neurons (here $n = 100$)

Parametric Space Search

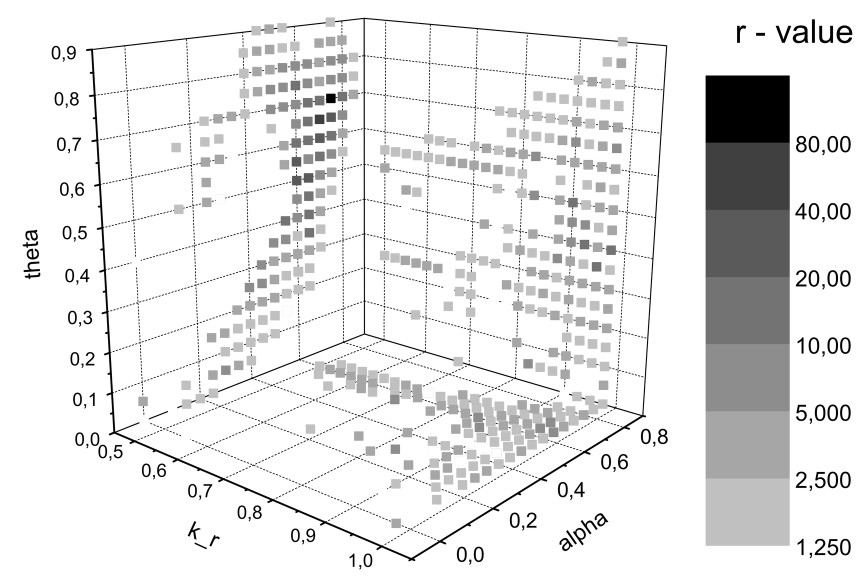
3-Dimensional parametric space (k_r, α, θ) is searched in order to find network chaotically exploring all state space. A statistical function r is introduced:

$$r(r_p) = \frac{average(r_p)^{\frac{3}{2}}}{deviation(r_p)}, \qquad (6)$$

- r_p is a set consisting of r_1, r_2, r_3, r_4
- r_i is a number of Hamming distance ≈ 0 events

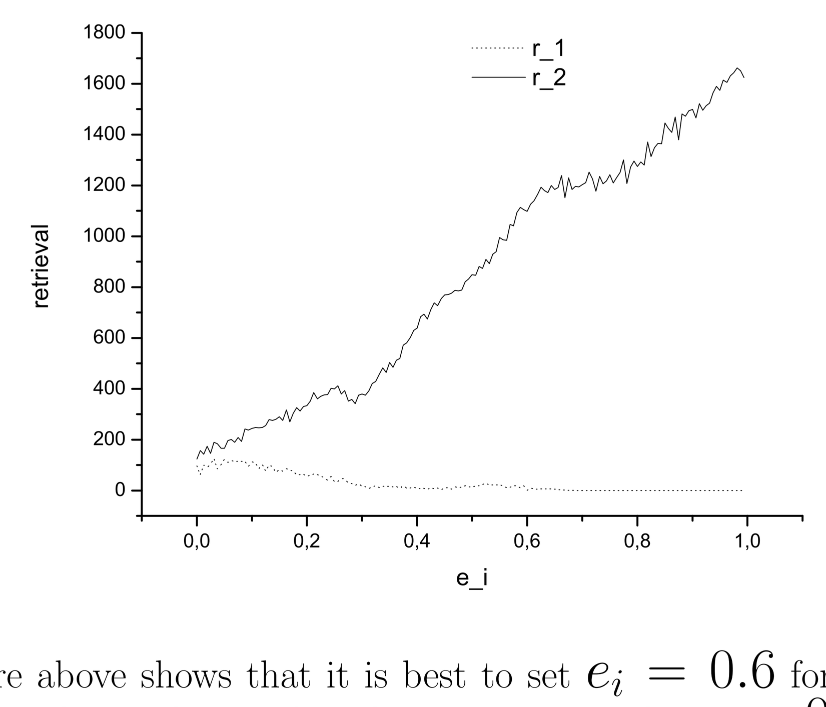
For every point in the (k_r, α, θ) space r_p value is assembled during 3000 iterations. Other parameters of the network are fixed for the whole search:

$k_a = k_r - 0.1$, $\varepsilon = 0.015$ and $e_i = 0$ for all i.



As depicted in the figure above, the best result is at the point:
$k_r = 0.975$, $\alpha = 0.75$, $\theta = 0.7$ and $k_a = 0.875$.
These parameters are used for further search.

The next step is to find optimal e_i value. In order to establish this value, r_1 and r_2 values are evaluated for every 3000 iterations with changing parameter e_i, where i is a serial number of a neuron with an external input embedded corresponding to the pattern 2.



The figure above shows that it is best to set $e_i = 0.6$ for neurons with external stimulus at the ith constituent and $e_j = 0$ with no external input at the jth constituent.

Results

With the set of all parameters fixed at optimal values after the searching phase responses and the dynamic of the network are investigated for presentations of single and mixed input patterns.

Input	retrieval (r_i)	LLE
no input	63 + 73 + 20 + 47	≈ 0.475
pattern 1	334 + 0 + 0 + 0	≈ 0.593
pattern 2	0 + 324 + 0 + 0	≈ 0.570
pattern 3	0 + 0 + 222 + 0	≈ 0.612
pattern 4	0 + 0 + 0 + 139	≈ 0.635
not stored	0 + 0 + 0 + 0	≈ 0.592
pattern 1+2	28 + 43 + 0 + 0	≈ 0.563
pattern 1+3	76 + 0 + 47 + 0	≈ 0.562
pattern 1+4	88 + 0 + 0 + 7	≈ 0.562
pattern 2+3	0 + 100 + 76 + 0	≈ 0.574
pattern 2+4	0 + 20 + 0 + 36	≈ 0.562
pattern 3+4	0 + 0 + 50 +12	≈ 0.572

The table above shows results after 4000 iterations.

Conclusions

The novel associative chaotic neural network introduced in this paper has been used to separate mixed binary input patterns into components of the stored patterns. The retrieval properties of the network are efficient and possible to improve. Although large Lyapunov exponents are present chaotic neural network can obtain high retrieval characteristics in the case of mixed external inputs.

Further investigation should focus on learning capabilities of the network. The significant difference between retrieval characteristics of the network in the case of stored and unknown patterns can be used for effective learning. Another issue that should be discussed is the separation of mixtures of more than two patterns.

One Step Further

It is possible to improve the separation ability of a chaotic network model. For that purpose a slight modification of equations 1 and 2 is proposed:

$$\eta_i(t+1) = k_a \eta_i(t) + \sum_{w_{ij} \in W_i^E} w_{ij} x_j(t) + e_i^{pos} \qquad (7)$$

$$\zeta_i(t+1) = k_r \zeta_i(t) - \alpha x_i(t) + + \sum_{w_{ij} \in W_i^I} w_{ij} x_j(t) + \theta - e_i^{neg} \qquad (8)$$

When external input is applied to the ith neuron, then $e_i^{pos} > 0$ and $e_i^{neg} = 0$, and when no input is applied to jth neuron, then $e_j^{pos} = 0$ and $e_j^{neg} > 0$. After searching parametric space (e^{pos}, e^{neg}) it is possible to find optimal value: $e_i^{pos} = 0.2$, $e_i^{neg} = 0$ when an input is added to the ith neuron and $e_j^{pos} = 0$, $e_j^{neg} = 0.15$ when there is no input to the jth neuron. Example of this dynamical behavior, when the input corresponds to a mixture of three patterns 1+2+3, is at the beginning of this poster ($r_1 = 137, r_2 = 114, r_3 = 112, r_4 = 0$).

Acknowledgements: Special thanks to Prof. Włodzisław Duch for his help in preparation of the article.