Chaotic Itinerancy for Patterns Separation

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Abstract. Chaotic neural network with external inputs has been used as a mixed input pattern separator. In contrast to previous work on the subject, highly chaotic dynamical system (LLE $\approx$ 0.6) is applied here. The network is based on a "dynamical mapping" scheme as an effective framework for cortical mapping. This feature allows for a more effective pattern retrieval and separation by the network.

1 Introduction

A chaotic neuron model was proposed by Aihara et. al [1]. Such neurons have been used to construct chaotic associative memory model [2]. External inputs corresponding to one of the embedded patterns force the network to show this pattern with higher frequency rate than other patterns. Nearly zero largest Lyapunov exponents were found. This kind of network is also able to separate mixed input patterns [3]. The network state wonders between the patterns present in the mixed input, occasionally retrieving other patterns. In this case largest Lyapunov exponents was negative.

In [4] a basic framework for the distributed coding scheme called *dynamical map* has been presented. It is realized by itinerancy among dynamical attractors of the network. This framework, which is still being developed, is used for simulating various associative memory phenomena [5], [6]. The main concept is based on the fact that external input corresponding to one of the stored patterns should drive the network from global chaotic attractor to nearly periodic local attractor lying in the basin of attraction of this stored pattern.

This paper is based on a combination of these two approaches. A novel chaotic associative memory is introduced in order to obtain Lyapunov exponents that are higher then those already reported [3], [1], [2]. A slight modification of the chaotic neural network developed previously has better retrieval and separation abilities. To prove this numerical simulations are presented.

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2 Network Model and Analysis Methods

In 1990 ([1]), the following chaotic neuron model in a n-neurons network was presented: $x_i(t+1) = f\left[\sum_{j=1}^{n} w_{ij} \sum_{d=0}^{t} k_f^d \, x_j(t-d) - \alpha \sum_{d=0}^{t} k_r^d g\{x_i(t-d)\} - \vartheta_i\right]$, where $x_i(t+1)$ denotes output of the ith neuron and f stands for a continuous output function. This equation, used also in [3], [2], is simplified and modified here to the following equations:

$$\eta_i(t+1) = k_a \eta_i(t) + \sum_{w_{ij} \in W_i^E} w_{ij} x_j(t) + e_i, \tag{1}$$

$$\zeta_i(t+1) = k_r \zeta_i(t) - \alpha x_i(t) + \sum_{w_{ij} \in W_i^I} w_{ij} x_j(t) + \theta, \tag{2}$$

$$\theta \equiv \vartheta_i (1 - k_r) \quad \text{for} \quad \text{all} \quad i, \tag{3}$$

$$x_i(t+1) = f\{\eta_i(t+1) + \zeta_i(t+1)\}. \tag{4}$$

$$f(u) = \frac{1}{1 + \exp(\frac{-u}{\varepsilon})}. \tag{5}$$

Here, $\eta_i(t)$ denotes all positive post-synaptical potentials and $\zeta_i(t)$ all negative potentials. This makes $\zeta_i(t)$ a resting potential, $\eta_i(t)$ a *pseudo*-action potential and $\eta_i(t) + \zeta_i(t)$ an action potential. Constants k_r and k_a are decay parameters for resting and *pseudo*-action potentials, respectively. The remaining parameters are: e_i - strength of the external input applied to the ith neuron, α - a refractory scaling parameter, ϑ_i - a threshold value of a chaotic neuron model, whereas θ - a threshold value of a simplified neuron model independent of the serial number i of a neuron, and finally ε - a steepness parameter of the continuous output function f. When action potential is positive, f function generates an output signal $x(t+1)$. The sets W_i^E and W_i^I consist of excitatory and inhibitory weights, respectively. They have been obtained from the Hebbian learning rule:

$$w_{ij} = \frac{1}{n} \sum_{p=1}^{m} (2x_i^p - 1)(2x_j^p - 1), \tag{6}$$

where x^p are the memorized patterns, for example the m=4 patterns shown in Fig. 1. The number of synchronously updated neurons in the simulations is set at $n = 100$, as in the previous work. Therefore equations 4-5 define a 200-dimensional discrete dynamical system.

Moreover, in the case of mixed patterns, the same technique as in [3] was used: for any two stored patterns logical OR (+) operation is performed. Mixed pattern composed of these two patterns is obtained. There are six possibilities of mixing the pairs of four patterns: 1+2, 1+3, 1+4, 2+3, 2+4, 3+4.

In order to evaluate retrieval characteristics of the network, the Hamming distance is used. This measures difference between spatio-temporal output and

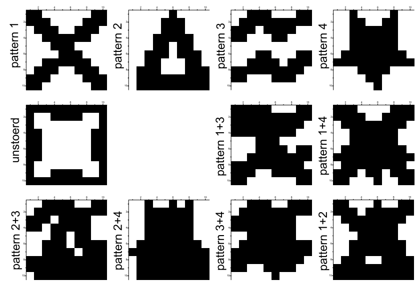

Fig. 1. Four stored patterns (1, 2, 3, 4), one additional pattern and six mixed patterns (1+3, 1+4, 2+3, 2+4, 3+4, 1+2) used in simulations of the associative chaotic neural network. 100-dimensional vectors are displayed in the form of a 10×10 matrix; white means 0 and black 1.

the stored patterns:

$$\mathrm{Ham}_p(x(t)) = \frac{1}{n}\sum_{i=1}^{n} |h(x_i(t)) - x_i^p|, \tag{7}$$

where p ($p = 1 - 4$) is a p-th pattern and h is the threshold function: $h(x) = \Theta(x - 0.5)$

During the simulation, it is possible to evaluate *conditional* retrieval characteristic r_p for every p-th pattern, that is the number of $\mathrm{Ham}_p(x(t)) \leq 0.5$ events. This condition helps to count every trajectory that is in the neighborhood of one of the stored patterns. With chaotic itinerancy the trajectory does not stay in the equilibrium, but wonders around the desired embedded patterns. To evaluate efficiency of the network the number of $\mathrm{Ham}_p(x(t)) = 0$ events are calculated. The number of these events is called an *exact* retrieval characteristic. The largest Lyapunov exponent is also used for evaluating dynamical properties of the network. It is evaluated from a Jacobian matrix and the Gram-Schmidt orthonormalization.

3 Parametric Space Search and Network Responses

First, 3-dimensional parametric space (k_r, α, θ) is searched in order to find network chaotically exploring all state space. A statistical function r is introduced:

$$r(r_p) = \frac{average(r_p)^{\frac{3}{2}}}{deviation(r_p)}, \tag{8}$$

where r_p is a set consisting of r_1, r_2, r_3, r_4. For every point in the (k_r, α, θ) space every r_p value is assembled during 2000 iterations. Initial conditions of the neural network, which are near the pattern 1, and all r_p values are reset each time the next point from the parametric space is chosen. Other parameters of the network are fixed for the whole search: $k_a = k_r - 0.1$, $\varepsilon = 0.015$ and $e_i = 0$ for all i. High value of the function r (equation 8) means that in the case of no external inputs, the network visits all stored patterns (high average) with almost the same frequency (low deviation). If for some point $[k_r, \alpha, \theta]$ $average(r_p) > deviation(r_p)$ and $r > 50$ then this point can constrain the neural network to the chaotic itinerancy state. As depicted in the figure 2, the best result ($r(r_p) =$

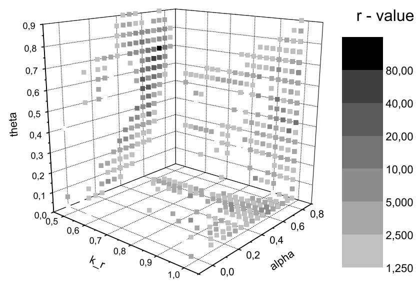

Fig. 2. Projections on the planes XY, YZ, XZ of the result of searching the parametric space (k_r, α, θ) for the best value of r. The $average(r_p)$ and $deviation(r_p)$ values are obtained during 2000 iterations.

86.26) is at the point $k_r = 0.975$, $\alpha = 0.75$, $\theta = 0.7$ and $k_a = 0.875$. These parameters are used for further search.

The next step is to find optimal e_i value. In order to establish this value, conditional retrieval characteristic r_1 for pattern 1 and r_2 for pattern 2 are evaluated for every 2000 iterations with changing parameter e_i, where i is a serial number of a neuron with an external input embedded corresponding to the pattern 2. The figure 3 shows that it is best to set $e_i = 0.6$ for neurons with external stimulus and $e_i = 0$ with no external input. Higher values of e_i can prevent chaos and pattern separation and lower values let the network visit other embedded pattern‘s basins.

With the set of all parameters fixed at optimal values after the searching phase responses of the network are investigated for presentations of single (table 1) and mixed (table 2) input patterns. The external input corresponds to one of the 100-pixel binary patterns or the logical OR mixture of these patterns shown in the figure 1. Table 1 shows the retrieval abilities of the network. The chaotic itinerancy among all embedded patterns and reverse patterns can be seen in the case of no input. When a single pattern is applied as an external stimulus the network stays in local chaotic attractor embedded near the desired pattern (see Fig. 4). For pattern that has not been stored nothing is retrieved, only occasionally reverse pattern 1 and 4 is retrieved because this pattern is slightly similar to them. Values of largest Lyapunov exponents are also given.

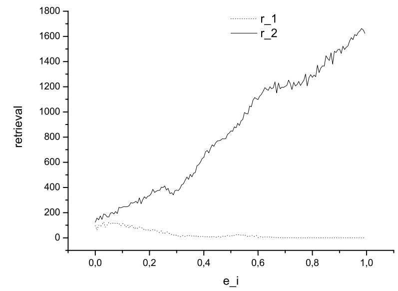

Fig. 3. Dependence of conditional retrieval characteristics for pattern 1 and 2 from on the changed parameter e_i.

Table 1. Retrieval ability: exact retrieval characteristics during 4000 iterations for every pattern and its reverse (in brackets). The largest Lyapunov exponents are also given.

Input	pattern 1	pattern 2	pattern 3	pattern 4	LLE
no input	63 (67)	73 (115)	20 (22)	47 (119)	≈ 0.475
pattern 1	334 (0)	0 (0)	0 (0)	0 (0)	≈ 0.593
pattern 2	2 (0)	324	0 (0)	2 (4)	≈ 0.570
pattern 3	0 (0)	0 (0)	222 (0)	0 (0)	≈ 0.612
pattern 4	0 (0)	0 (0)	0 (0)	139 (0)	≈ 0.635
unstored	0 (15)	0 (0)	0 (0)	0 (21)	≈ 0.592

Table 2 shows the separation ability of the chaotic network. No patterns other than components of the mixed pattern are retrieved. Conditional and exact retrieval characteristics prove that the trajectory of investigated system wonders chaotically around these components. Presented network visits other patterns only at the transition phase, which is the first several dozen iterations (see figure 4). High (but rather similar) values of largest Lyapunov exponents should be noted.

4 Conclusions

The novel associative chaotic neural network introduced in this paper has been used to separate mixed binary input patterns into components of the stored patterns. The retrieval properties of the network are more efficient than found in the previous reports [3], [2]. Although large Lyapunov exponents are present chaotic neural network can obtain high exact and conditional retrieval characteristics in the case of mixed external inputs.

Further investigation should focus on learning capabilities of the network. The significant difference between retrieval characteristics of the network in the

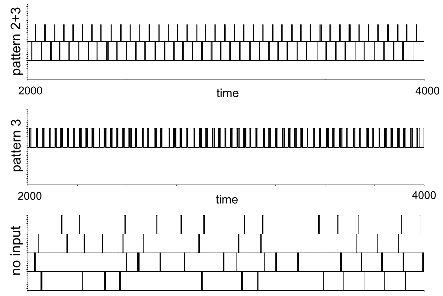

Fig. 4. Dynamical behavior of the network in the case of mixed, single and no pattern input. Temporal variation of the Hamming distances $\mathrm{Ham}_p(t)$ $(p = 1-4)$. When $Ham_p \leq 0.5$, a vertical bar is drawn on pth row.

Table 2. Separation ability: retrieval characteristics during 4000 iterations for every pattern: the exact value and conditional value (in brackets). The largest Lyapunov exponents are also given.

Input	pattern 1	pattern 2	pattern 3	pattern 4	LLE
pattern 1+2	28 (168)	43 (345)	0 (0)	0 (0)	≈ 0.563
pattern 1+3	76 (400)	0 (0)	47 (344)	0 (0)	≈ 0.562
pattern 1+4	88 (394)	0 (0)	0 (6)	7 (60)	≈ 0.562
pattern 2+3	0 (2)	100 (366)	76 (375)	0 (0)	≈ 0.574
pattern 2+4	0 (0)	20 (268)	0 (0)	36 (220)	≈ 0.562
pattern 3+4	0 (3)	0 (0)	50 (410)	12 (105)	≈ 0.572

case of stored and unknown patterns can be used for effective learning. Another issue that should be discussed is the separation of mixtures of more than two patterns.

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